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Non-standard compactifications with mass gaps and Newton's law

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Abstract: The four-dimensional Minkowski space-time is considered as a three-brane embedded in five dimensions, using solutions of five-dimensional supergravity. These backgrounds have a string theoretical interpretation in terms of D3-brane distributions. By studying linear fluctuations of the graviton we find a zero-mode representing the massless graviton in four-dimensional space-time. The novelty of our models is that the graviton spectrum has a genuine mass gap (independent of the position of the world-brane) above the zero-mode or it is discrete. Hence, an effective four-dimensional theory on a brane that includes the massless graviton mode is well defined. The gravitational force between point particles deviates from the Newton law by Yukawa-type corrections, which we compute explicitly. We show that the parameters of our solutions can be chosen such that these corrections lie within experimental bounds.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Supergravity Models.

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Contents

1. Introduction

The idea that our four-dimensional Minkowski space-time M_4 can be viewed as a three-brane embedded in some higher-dimensional curved space-time is appealing and attracted attention already some years ago [1]. It was recently revived in several works. In particular in [2, 3] an alternative to the resolution of the mass hierarchy problem between the Planck ($M_{\mathrm{Pl}} \sim 10^{19}$ GeV) and electroweak ($M_{\mathrm{E}} \sim 1$ TeV) scales using extra large dimensions [4] was presented. Indeed, in the latter scenario a compactification on a general compact manifold B_n with $n \geq 2$ requires that the size of a typical extra dimension be of order $R = 1$ mm or smaller, which is in principle accessible to experiments in the near future (see, for instance, [5] and references therein). However, as far as the hierarchy problem is concerned, the original mass hierarchy of $M_{\mathrm{Pl}}/M_{\mathrm{E}} \sim 10^{16}$ is replaced by a new one of the same order of magnitude between the electroweak scale and the size of the large dimensions. In [2, 3] a model based on a slice of the five-dimensional anti-de Sitter space (AdS_5) was proposed. The effective four-dimensional Planck constant was determined by the curvature of the embedding space rather than the size of the extra dimension, which is of the same order as the fundamental five-dimensional scale. Assuming that the square of the fundamental five-dimensional mass scale and the curvature of AdS_5 are of the same order of M_{Pl}, one shows that the required hierarchy is of the order of 10^2.

The scenario of [2, 3] has the desired feature that there exists a square-normalizable state representing a massless graviton. However, there is in addition a continuum of massive modes with no mass gap separating them from the massless one. If the curvature of AdS_5 is of the order of the four-dimensional effective Planck scale, then this gives no measurable effect to modifications of, for instance, the Newton law [3]. However, given our present-day experimental data, this might not be the case and such corrections may exist [5]. Moreover, in order to have a well-defined effective field theory, it is desirable to have solutions where the massless graviton is separated from the other massive modes by a mass gap. These massive modes can be continuous or discrete, with no immediate effect in phenomenological considerations. It is the purpose of this paper to construct such models.

It is also desirable to have a string theoretical construction for our models. The model in [2, 3] can be thought of as a compactification of the near-horizon limit of the solution for a large number of coinciding D3-branes in type-IIB string theory on S^5, and a subsequent truncation of the range of the extra fifth dimension. The background in [3] with one three-brane shares many features with domain wall solutions of four-dimensional supergravity theories, which were studied extensively in the literature [6]. A geometry similar to that of the set-up in [2] appears in strongly coupled heterotic string theories, which arise in Calabi–Yau compactifications of the Horava–Witten model [7]. In [8] a domain wall solution with two three-branes at the two boundaries was constructed, which is suitable for a further reduction to four-dimensional supergravity. In [9] a ten-dimensional background of a configuration of D3-branes in an orientifold of type-II string theory was presented, which interpolates between $AdS_5 \times S^5$ and $M_4 \times T^6$ and contains four-dimensional gravity on the branes. It is closely related to the set-up in [2, 3] and can be viewed as an extension of the AdS/CFT correspondence [10] to boundary theories including gravity. It is obvious [11] that the scenario of [2, 3] applies to all minima of the potential of the five-dimensional gauged supergravity [12], supersymmetric or not, although the latter minima may not be stable (at least perturbatively).

In this paper we consider backgrounds of continuous distributions [13, 14] of D3-branes of type-IIB string theory in the near horizon limit, which preserve sixteen supersymmetries. We will consider in detail two examples: one that represents D3-branes uniformly distributed over a disc and one where the distribution is over a three-sphere. These examples were studied before in [15, 16] in connection with the Coulomb branch of strongly coupled $N = 4$ super Yang-Mills theories within the AdS/CFT correspondence. They possess many of the desired features we would like to have, in particular they have a mass gap or a discrete spectrum for the case of the disc and sphere distributions, respectively. However, the massless mode is not square normalizable and this is is one issue we address and solve in this paper. Although we will concentrate on these two backgrounds for concreteness, we believe that one can in this context study a much larger class of distributions of D3-branes e.g. shell-type distributions which were also studied before in [17] as models for the Coulomb branch of $N = 4$ super Yang-Mills.

The outline of the paper is as follows: in section 2 we derive the five-dimensional backgrounds in the presence of a three-brane that cuts off the space in the fifth direction. In section 3 we study the spectrum of linear fluctuations of the graviton in these backgrounds. It turns out that the equation for gravitons polarized in the directions parallel to the brane is identical to the massless scalar equation in the same backgrounds. We will show that a normalizable zero-mode solution, representing the massless graviton, exists, whereas the massive modes are separated by a mass gap. We use these models in section 4 to study possible measurable effects manifested as corrections to Newton's law, which turn out to be of Yukawa-type. Our analysis has certain similarities with that performed for the case of compactifications on general compact manifolds in [18]. The range of the exponential correction is associated with the wavelength of the lightest massive state, whereas its strength is related to the value of the corresponding wave function at the position of the brane. In section 5 we present our conclusions and some future directions of this work. We have also written an appendix, containing some details of the relation between ten- and five-dimensional backgrounds and the corresponding five-dimensional gauged supergravity theories.

2. The models

In the following we exploit the fact that there exists a big class of ten-dimensional backgrounds, besides $AdS_5 \times S^5$, that can be reduced to five-dimensional models. After the reduction to five dimensions the backgrounds become warped products of four-dimensional Minkowski space with one extra dimension. Then we reduce the range of this coordinate at some finite value and place a three-brane at the boundary.[1] Although the fifth coordinate is not necessarily compact, these backgrounds are all effectively compactifications, since the Kaluza–Klein spectrum contains a zero-mode corresponding to the four-dimensional gravity of our world and there exists also a mass gap. This mass gap can be chosen such that the corrections to Newton's law lie within present-day experimental limits. Therefore, an observer on the brane will effectively see a four-dimensional world.

We will concentrate on two backgrounds: one (to be called (A)) corresponds to D3-branes distributed uniformly over a disc, with metric given by

$$ds^2 = \frac{r^{4/3}}{R^2}(r^2 + r_0^2)^{1/3}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^{2/3}}(r^2 + r_0^2)^{-2/3}dr^2\,, \qquad r \geq 0\,, \qquad (2.1)$$

[1]The important issue of the determination of the location of the brane from first principles will not be addressed in this paper. For the model in [2, 3] a suggestion, based on a modulus-field stabilization mechanism, was made [19]; it can presumably be modified to cover our models as well.

and one (to be called (B)) corresponding to D3-branes distributed uniformly over a three-sphere, with metric given by

$$ds^2 = \frac{r^{4/3}}{R^2}(r^2 - r_0^2)^{1/3}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^{2/3}}(r^2 - r_0^2)^{-2/3}dr^2\,, \qquad r \geq r_0\,. \tag{2.2}$$

Note that the two metrics are related by the analytic continuation $r_0 \to ir_0$ and $\eta_{\mu\nu}$ is the metric of four-dimensional Minkowski space-time.

It is useful to present the metrics in their conformally flat form

$$ds^2 = e^{2\Omega(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\,. \tag{2.3}$$

For model (A) we find that the coordinate transformation

$$r = \frac{r_0}{\sinh(z/R)}\,, \qquad z \geq 0\,, \tag{2.4}$$

and a rescaling of the space-time coordinates as $x^\mu \to \frac{R}{r_0}x^\mu$ transforms (2.1) into the form (2.3) with conformal factor

$$e^\Omega = \frac{\cosh^{1/3}(z/R)}{\sinh(z/R)}\,. \tag{2.5}$$

Similarly for model (B), the coordinate transformation

$$r = \frac{r_0}{\sin(z/R)}\,, \qquad 0 \leq z \leq \frac{\pi R}{2}\,, \tag{2.6}$$

and the same rescaling of the x^μ's as before, transforms (2.2) into the form (2.3) with conformal factor

$$e^\Omega = \frac{\cos^{1/3}(z/R)}{\sin(z/R)}\,. \tag{2.7}$$

Near the boundary $r \to \infty$ ($z \to 0$) both spaces are asymptotic to AdS_5. Equivalently, we recover AdS_5 if we let the parameter R go to infinity.

Our models are constructed by taking the above backgrounds and cutting out the boundary region $0 \leq z \leq z_1$ by placing a three-brane at $z = z_1$ (compare footnote 1). This restriction on the range of z is necessary to obtain dynamical gravity in the effective four-dimensional theory. It is useful to consider the double cover of this space, which amounts to requiring reflection symmetry with respect to the three-brane. A feature of these backgrounds is that the five-dimensional bulk action also contains a scalar field with a potential (some facts about five-dimensional supergravity relevant to our backgrounds are summarized in appendix). The five-dimensional action is the sum of a bulk term and a boundary term:

$$\begin{aligned} S &= S_{\text{bulk}} + S_{\text{brane}} \\ &= \int_{z_1}^{z_{\max}} dz \int d^4x\sqrt{-G}\left(\frac{1}{4}\mathcal{R} - \frac{1}{2}G^{MN}\partial_M\phi\partial_N\phi - P(\phi)\right) + \\ &\quad + \int d^4x\sqrt{-\widetilde{G}}(\mathcal{L}_{\text{brane}} - V)\,, \end{aligned} \tag{2.8}$$

with $z_{\max} = \infty$ in case (A) and $z_{\max} = \pi R/2$ in case (B). The five-dimensional metric is denoted by G_{MN}, and $\mathcal{R}$ is the corresponding Ricci scalar. The potential P of the scalar field ϕ is defined in appendix, and $\widetilde{G}$ is the pullback of the bulk metric to the four-dimensional world volume of the three-brane. The brane action contains a term, $\mathcal{L}_{\text{brane}}$, that corresponds to the matter fields living on the brane and will be ignored in the following; the other term is V, which corresponds to the tension of the brane and which will be fixed shortly by requiring consistency of the equations of motion. We will show that this also implies the vanishing of the effective cosmological term on the three-brane.

The equations of motion following from varying the metric and the scalar field in the action (2.8) are

$$\begin{aligned} &\frac{1}{4}R_{MN} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{1}{3}PG_{MN} - T_{MN} + \frac{1}{3}T_L^L G_{MN} = 0\,, \\ &\partial_M\left(\sqrt{-G}G^{MN}\partial_N\phi\right) - \sqrt{-G}\frac{\partial P}{\partial\phi} - \sqrt{-\widetilde{G}}\frac{\partial V}{\partial\phi}\delta(z-z_1) = 0\,, \end{aligned} \tag{2.9}$$

where the energy–momentum tensor T_{MN} of the source term coming from the three-brane is

$$T_{MN} = -\frac{V}{2}\frac{\sqrt{-\widetilde{G}}}{\sqrt{-G}}\widetilde{G}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(z-z_1)\,. \tag{2.10}$$

The solutions to these equations can be expressed in terms of the bulk solutions without the three-brane, by a simple replacement that preserves reflection invariance $z - z_1 \to -(z - z_1)$ with respect to the brane

$$z \to \tilde{z} = z_1 + |z - z_1|\,. \tag{2.11}$$

This introduces terms proportional to $\delta(z - z_1)$ in the equations of motion, which are cancelled by the source term if we choose the vacuum energy of the three-brane to be

$$V = -3\Omega'(z)\exp(-\Omega) = \frac{2}{R}W(\phi)\,, \tag{2.12}$$

where W is an auxiliary function of the scalar field ϕ and is defined in appendix.

We will focus our attention to solutions of (2.9) that preserve some supersymmetry, so that the corresponding backgrounds are stable, at least perturbatively. The Killing spinor conditions for preserving sixteen supercharges, in the absence of the brane at $z = z_1$, were derived in [15] and are summarized in appendix. In our case it turns out that they are slightly altered by the presence of the brane

$$\begin{aligned} \frac{d\Omega}{dz} &= -\frac{2}{3R}e^\Omega W \times \text{sign}(z-z_1)\,, \\ \frac{d\phi}{dz} &= \frac{1}{R}e^\Omega\frac{\partial W}{\partial\phi} \times \text{sign}(z-z_1)\,. \end{aligned} \tag{2.13}$$

The solution is given by the bulk solution after replacing $z \to \tilde{z}$ in (2.5) and (2.7).

Now that we have found the backgrounds, we can proceed to calculate several properties of the effective four-dimensional models. In this paper we will concentrate on the gravity sector. This is partly because we do not know the exact form of $\mathcal{L}_{\rm brane}$; of course, we could impose the standard model lagrangean, or try to derive it from first principles. In the second case, keeping in mind the string theory origin of our backgrounds, this would most probably amount to expanding the Born–Infeld action in the corresponding background. We leave this for future studies and ignore for the moment contributions of the matter sector to the gravity equations. The quantities we want to study in the following are the four-dimensional Planck constant and the four-dimensional cosmological constant, which should vanish for a physical model. In section 3 we will discuss the spectrum of linearized graviton fluctuations and in section 4 the corrections to Newton's law.

In order to calculate the four-dimensional Planck mass in terms of the five-dimensional fundamental scale, we express the five-dimensional metric G_{MN} in terms of a four-dimensional metric $G_{(4)\mu\nu}$, which replaces $\eta_{\mu\nu}$ in (2.3) so that the various four-dimensional geometrical data are non-vanishing. The effective four-dimensional lagrangean thus derived from (2.8) is

$$\mathcal{L}_{4d} = M^3 \int_{z=z_1}^{z_{\rm max}} \sqrt{-G_{(4)}} \left(\frac{1}{4} e^{3\Omega} R_{(4)} - 3e^{3\Omega}(\Omega')^2 - 2e^{3\Omega}\Omega'' - \frac{1}{2} e^{3\Omega}(\phi')^2 - \right. \\ \left. - e^{5\Omega} P + \frac{3}{2}\Omega' e^{3\Omega}\delta(z-z_1) \right) , \tag{2.14}$$

where $z_{\rm max} = \infty$ in case (A) and $z_{\rm max} = \pi R/2$ in case (B), and M is the five-dimensional fundamental Planck constant. The effective Planck constant in four dimensions, $M_{\rm Pl}$, can be read off from the first term in (2.14). In case (A), we find

$$M_{\rm Pl}^2 = M^3 \int_{z_1}^{\infty} dz\ e^{3\Omega(z)} = \frac{M^3 R}{2} \frac{1}{\sinh^2(z_1/R)} , \tag{2.15}$$

whereas in case (B)

$$M_{\rm Pl}^2 = M^3 \int_{z_1}^{\pi R/2} dz\ e^{3\Omega(z)} = \frac{M^3 R}{2} \cot^2(z_1/R) . \tag{2.16}$$

Because of the non-vanishing scalar potential $P(\phi)$, the five-dimensional model has an effective negative cosmological constant that is z-dependent. Hence, it is not obvious that this will not induce an undesirable cosmological constant in four dimensions (the latter, if non-vanishing, can be consistently set to zero for the purposes of this paper). However, this is prevented even by mathematical consistency, since it would imply that the metric on the three-brane would not be a Minkowski but a curved one. Indeed it may be checked that although all terms in (2.14), except the first one, contribute to the cosmological constant, the final result is zero.

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2.1 The parameter space

Our models have three parameters: the fundamental five-dimensional mass scale M, the position of the brane z_1, and the length parameter R. These have to be chosen in such a way that supergravity is a good approximation as an effective theory. A straightforward computation for the scalar curvature for the metric (2.3) gives

$$\mathcal{R} = -4e^{2\Omega}\left(2\Omega'' + 3\Omega'^2\right). \tag{2.17}$$

Using the explicit expressions for the conformal factors (2.5) or (2.7), it turns out that for ratios z/R not too close to $z_{\rm max}$, the scalar curvature becomes $\mathcal{R} \sim -\text{const.}/R^2$, where the proportionality constant is of order 1. A similar statement holds for the other curvature invariants. Hence, the condition $|\mathcal{R}| \ll M^2$ for supergravity to be valid reduces to

$$RM \gg 1\,. \tag{2.18}$$

Keeping this in mind, we investigate the different physical pictures that are obtained from various choices of the location of the three-brane z_1, as well as of the fundamental five-dimensional scale M and the length parameter R.

Case I. The three-brane is located at the point where

$$e^{\Omega(z_1)} = 1\,, \tag{2.19}$$

which implies, using the background (2.3), that $z_1 = 0.984R$ when the conformal factor is given by (2.5) and $z_1 = 0.972R$ when the conformal factor is given by (2.7). We take the radius of curvature of our background to be of the order of 1 mm and find that

$$R \sim 1\,\text{mm} \sim 10^{14}\,\text{GeV}^{-1}\,, \qquad M \sim 10^{8}\,\text{GeV}\,, \tag{2.20}$$

where the five-dimensional fundamental mass scale was found using (2.15). Notice that this value is intermediate between the Planck and electroweak energy scales. The picture that emerges is similar to the case of compactification on a large torus of radius $\sim 1\,$mm. Consistent with that is the fact that both (2.15) and (2.16) can be approximated by

$$M_{\rm Pl}^2 \sim M^3 R\,. \tag{2.21}$$

Similarly to [4] (for the case of compactification on a large torus), unification of the fundamental scale M and the electroweak scale $M_{\rm E}$ cannot be achieved unless $R \sim 10^{13}\,$m, i.e. R has astronomical size. Choosing M at an intermediate scale, as in (2.20), we avoid this problem and we are consistent with present-day data.

Case II. The three-brane is located at the point where

$$e^{\Omega(z_1)} \sim \frac{R}{z_1} \sim 10^{11}\,, \tag{2.22}$$

and M and $1/z_1$ are both taken to be of the order of the electroweak scale $\sim 1\,\text{TeV}$. Since the argument of the conformal factor is small the four-dimensional Planck constant can be approximated as

$$M_{\text{Pl}}^2 \sim \frac{M^3}{z_1^2} R^3 \sim M^5 R^3 \,, \tag{2.23}$$

which is similar to compactifications with three extra dimensions of large size $R \sim 10^{-5}\,\text{mm} \sim 10^8\,(\text{GeV})^{-1}$. In this model the wave functions of the massive gravitons are extremely small, at $z = z_1$. For this reason, as we shall see in section 4, the Yukawa-type corrections to Newton's law turn out to be negligible.

We note that for the choice of parameters we have made above, the condition (2.18) is clearly satisfied. A feature that the two geometries share is that the curvature blows up close to the brane distribution (at $z \to \infty$ and $z \to \pi/2R$ for model (A) and (B) respectively) and corrections due to higher-derivative terms in the action become important. However, the region of large curvature is very small and the qualitative picture will not be altered at all. This point was discussed in [15, 16] from the point of view of the ten-dimensional D3-brane solution.

3. The graviton: massless and massive modes

In this section we study small fluctuations $h_{\mu\nu}$ of the four-dimensional Minkowski metric on the brane and determine the graviton spectrum. In general, this will depend on the details of the five-dimensional backgrounds where the four-dimensional Minkowski metric is embedded.

We parametrize the fluctuations in the following manner

$$ds^2 = e^{2\Omega(\tilde{z})} \left((\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dz^2 \right) , \tag{3.1}$$

as it is consistent to set to zero the components of the graviton fluctuations $h_{5\mu}$ and h_{55} as well as the fluctuations of the scalar field ϕ. Then we insert (3.1) into (2.9) and linearize in $h_{\mu\nu}$. The calculation is facilitated by the fact that the metric is conformally flat. We may also utilize the reparametrization invariance by choosing the gauge $h^\mu_\mu = \partial^\mu h_{\mu\nu} = 0$. In this gauge we find the following equation:

$$\Box_x h_{\mu\nu} + \partial_z^2 h_{\mu\nu} + 3\frac{d\Omega(\tilde{z})}{dz} \partial_z h_{\mu\nu} = 0 \,, \tag{3.2}$$

which is just the Laplace equation of a massless scalar in the five-dimensional background (3.1). The source term due to the three-brane cancels out completely, but the presence of the brane at $z = z_1$ demands that appropriate boundary conditions are chosen, as we shall see. We consider fluctuations that are plane waves in the four-dimensional Minkowski space-time

$$h_{\mu\nu}(x, z) = \exp(ik \cdot x) h_{\mu\nu}(z) \,. \tag{3.3}$$

Then we find the following simple differential equation (we can drop the indices from $h_{\mu\nu}$ since the differential equation is the same for all the components of the graviton):

$$h'' + 3\frac{d\Omega(\tilde{z})}{dz}h' + M^2 h = 0\,, \tag{3.4}$$

where $M^2 = -k \cdot k$ is the mass square of the corresponding graviton fluctuation mode. At the location of the three-brane, the wave function h and its first derivative are smooth functions. This implies, because of the reflection symmetry with respect to the three-brane, that the first derivative of h has to vanish at the location of the three-brane. This condition also guarantees the hermiticity of the laplacean in the curved background (3.1). It is obvious that there exists a massless mode

$$h_0 = \text{const.}\,, \tag{3.5}$$

that solves (3.1) and also has vanishing first derivative at the position of the brane at $z = z_1$. Moreover, it is normalizable in the interval $z_1 \leq z < z_{\text{max}}$, with measure $dze^{3\Omega}$. In the rest of this section we determine the spectrum of the massive graviton fluctuations for our two models.

3.1 Model A

We first determine the mass spectrum of graviton fluctuations for non-zero M for the model corresponding to a uniform distribution of D3-branes on a disc. In order to solve the differential equation we found it useful to change variable to

$$x = \frac{U^2}{U^2 + r_0^2} = \frac{1}{\cosh^2(z/R)}\,, \qquad 0 \leq x \leq x_1 \equiv \frac{1}{\cosh^2(z_1/R)} < 1\,. \tag{3.6}$$

Then the function $hx^{(1-q)/2}$ obeys a hypergeometric equation. Hence, we can easily write down the general solution for the graviton fluctuations as

$$h_q = \mathcal{N}_q\Big(e^{i\theta_q}\; x^{(q-1)/2} F_q(x) \;+\; e^{-i\theta_q}\; x^{-(q+1)/2} F_{-q}(x)\Big)\,. \tag{3.7}$$

The constant q and the function $F_q(x)$ are related to the mass M and a particular hypergeometric function as

$$\begin{aligned} F_q(x) &\equiv F\Big(\frac{q-1}{2}, \frac{q-1}{2}, 1+q; x\Big)\,, \\ q &\equiv \sqrt{1 - R^2M^2}\,, \end{aligned} \tag{3.8}$$

where $\mathcal{N}_q$ and θ_q are real constants. The above solution takes the same form as that given in [16], after using certain transformation properties of hypergeometric functions. The coefficients and the allowed values of M are now determined by requiring normalizability and the appropriate boundary conditions. Since x does not take values in the entire unit interval, the hermiticity condition for the scalar Laplace operator requires that

$$\partial_x h\big|_{x=x_1} = 0\,. \tag{3.9}$$

This determines the phase θ_q as

$$e^{2i\theta_q} = \left.\frac{(q+1)F_{-q}(x) - 2x\partial_x F_{-q}(x)}{(q-1)F_q(x) + 2x\partial_x F_q(x)}\right|_{x=x_1}\,. \tag{3.10}$$

Note that θ_q depends not only on the value of q, but also on the particular point x_1. As was shown in [16] (see also [15]) the parameter q in (3.8) is purely imaginary and therefore there is a mass gap in the spectrum

$$M_{\text{gap}} = \frac{1}{R}\,, \tag{3.11}$$

and, above it, there is a continuum with $M \geq 1/R$. This follows from requiring orthonormalizability in the Dirac sense (with the use of a δ-function). We also note that the existence of a mass gap is most transparent if we transform the equation for the graviton fluctuations $h(z)$ into a Schrödinger equation by means of the transformation $h(z) = e^{-3\Omega/2}\Psi(z)$

$$-\frac{d^2\Psi}{dz^2} + V(z)\Psi = M^2\Psi\,, \tag{3.12}$$

with potential

$$V(z) = \frac{3}{2}\Omega''(z) + \frac{9}{4}\Omega'^2(z) + 3\Omega'(z_1)\delta(z-z_1)\,. \tag{3.13}$$

In our case, using (2.5), we obtain

$$V(z) = \frac{1}{R^2}\left(1 + \frac{1}{4\cosh^2(z/R)} + \frac{15}{4\sinh^2(z/R)}\right) - \frac{2}{R}\frac{2+\cosh(2z_1/R)}{\sinh(2z_1/R)}\delta(z-z_1)\,, \tag{3.14}$$

which is a particular member of the class of potentials known in the literature as Pöschl–Teller potentials of type II. Clearly, since $V(z\to\infty) = 1/R^2$, there is a minimum value for M^2 given by (3.11). Note also that the value of the mass gap does not depend on the particular location of the brane at z_1. The reason is that the mass gap corresponds to the asymptotic value of the potential for large z. The zero-mode wave function is

$$\Psi_0(z) = \frac{\cosh^{1/2}(z/R)}{\sinh^{3/2}(z/R)}\,, \tag{3.15}$$

and is clearly normalizable in the interval $z_1 \leq z < \infty$. Of course it corresponds to (3.5) after we multiply with the factor $e^{3\Omega/2}$.

3.2 Model B

Let us now turn to the case of the model corresponding to D3-branes uniformly distributed over a three-sphere. In order to solve the differential equation (3.4) with (2.7) we found it useful to change variable to

$$x = 1 - \frac{r_0^2}{U^2} = \cos^2(z/R)\,, \qquad 0 \le x \le x_1 \equiv \cos^2(z_1/R) < 1\,. \tag{3.16}$$

Then the function $(1-x)^{-2}h$ obeys a hypergeometric equation. The general solution for the graviton fluctuations that is regular at $x = 0$ is given by

$$h_q = \tilde{\mathcal{N}}_q (1-x)^2 F(q+2, -q+1, 1; x)\,, \tag{3.17}$$

where $\tilde{\mathcal{N}}_q$ is a normalization constant and the real number q parametrizes the mass M as $M = \frac{2}{R}\sqrt{q(q+1)}$. The hermiticity condition for the scalar Laplace operator is equivalent to the condition (3.9) for the derivative of the graviton wave function. This determines the massive spectrum as

$$M_q = \frac{2}{R}\sqrt{q(q+1)}\,, \tag{3.18}$$

where q belongs to a discrete set of real numbers that can be taken to be positive with no loss of generality. Hence, as in the case of the previous model, there exists a mass gap, corresponding to the lowest eigenvalue for q, separating the massless graviton mode from the massive spectrum.[2] As before we may cast our eigenvalue problem into an equivalent Schrödinger problem. The corresponding potential is given by (3.13) after we use (2.7)

$$V(z) = \frac{1}{R^2}\left(-1 - \frac{1}{4\cos^2(z/R)} + \frac{15}{4\sin^2(z/R)}\right) - \frac{2}{R}\frac{2+\cos(2z_1/R)}{\sin(2z_1/R)}\delta(z-z_1)\,. \tag{3.19}$$

Similarly to (3.14), it is a particular member of the class of potentials known in the literature as Pöschl–Teller potentials of type I. The zero-mode wave function for the potential (3.19) is given by

$$\Psi_0(z) = \frac{\cos^{1/2}(z/R)}{\sin^{3/2}(z/R)}\,, \tag{3.20}$$

which is normalizable in the interval $z_1 \le z < \pi R/2$. Of course it corresponds to (3.5) after we multiply it by xsthe factor $e^{3\Omega/2}$. The Schrödinger differential equations for the potentials (3.14) and (3.19) are related by the analytic continuation $z \to iz$, as expected from a similar relation between the corresponding supergravity backgrounds.

[2]For $z_1 = 0.972R$ (equivalently $x_1 = 0.318$) we have $e^{\Omega(z_1)} = 1$ and the first few eigenvalues are $q = 2.74, 5.24, 8.00, 10.6\ldots$ As the brane position z_1 moves closer to $z = 0$ (equivalently $x = 1$) the corresponding eigenvalues move towards the set of positive integers (see (3.21) and (3.22) below). Hence, the mass gap corresponding to the lowest eigenvalue for q is insensitive, for all practical purposes, to the position of the brane.

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In general the wave functions (3.17) are not orthogonal for different values of q and in addition they overlap with the massless constant mode h_0. In principle it is straightforward to construct an orthonormal basis, although in practice one eventually resorts to numerical methods. There are, however, two particular choices for the position of the brane that lead to a great mathematical simplification and make the physical picture more transparent. In the first case, consider $x_1 \simeq 1$. This corresponds to $z_1 \ll R$ as in case II discussed in section 2.1. In this case, (3.17) (with q a positive integer) is related to Jacobi polynomials. The complete set of mutually orthogonal massive graviton wave functions that are normalized to 1 is practically the same as the one found in [16], where $x_1 = 1$ exactly. They read

$$h_n = \sqrt{\frac{2(2n+1)}{R}}(1-x)^2 P^{(2,0)}_{n-1}(2x-1)\,, \qquad 0 \le x \le 1\,, \qquad n = 1, 2, \dots\,, \quad (3.21)$$

where $P^{(2,0)}_{2n-1}$ are the appropriate Jacobi polynomials. The mass spectrum is, to an extremely good approximation, given by

$$M_n = \frac{2}{R}\sqrt{n(n+1)}\,, \qquad n = 1, 2, \dots\,. \quad (3.22)$$

Another case where Jacobi polynomials arise is for $x_1 = 1/2$ (which corresponds to $z_1 = \pi R/4 \sim 0.785R$ and $e^{\Omega(z_1)} = 2^{1/3} \sim 1.26$). Note that for this choice of the brane location we are effectively discussing a parameter choice similar to that made in case I in section 2.1. Here, the set of real numbers q in (3.17) coincides with the set of even integers. It turns out that the complete set of mutually orthogonal massive graviton wave functions that are normalized to 1 is

$$h_n = 2\sqrt{\frac{4n+1}{R}}(1-x)^2 P^{(2,0)}_{2n-1}(2x-1)\,, \qquad 0 \le x \le \frac{1}{2}\,, \qquad n = 1, 2, \dots\,, \quad (3.23)$$

whereas the mass spectrum is given by

$$M_n = \frac{4}{R}\sqrt{n(n+1/2)}\,, \qquad n = 1, 2, \dots\,. \quad (3.24)$$

The set of eigenfunctions (3.23) is by itself complete and all of its members are orthogonal (with respect to the usual inner product) to the constant massless mode. The same of course is true for (3.21).

4. The Newton law

The fact that we have modelled our flat four-dimensional space-time as a brane embedded in a five-dimensional curved space-time, has certain consequences for the Newton law that governs the gravitational attraction of point particles in our four-dimensional world. Present-day experimental data do not exclude corrections to the

$1/r^2$ attractive force for distances smaller than or equal to 1 mm. Since our models are five-dimensional at the fundamental level, we should, at distances much smaller than R, have the Newton law in four spatial dimensions

$$V = -\frac{G_5 M}{r_4^2}\,, \tag{4.1}$$

where r_4 denotes the corresponding radial distance from a point mass M located at the origin, and G_5 is the Newton constant in $4+1$ dimensions. On the other hand, for distances much larger than R, we should just obtain the usual Newton law in three spatial dimensions

$$V = -\frac{G_4 M}{r}\,, \tag{4.2}$$

where r is the usual radial distance in three spatial dimensions and G_4 is the Newton constant in $3+1$ dimensions. The crossover behaviour between (4.1) and (4.2) depends on the details of the particular model used. In particular, in view of possible experimental verifications, we are interested in computing exponential corrections to the leading-order behaviour of the Newton potential (4.2). At this point we emphasize again the importance of the existence of a mass gap in our models, as this will govern the behaviour of the leading Yukawa-type correction to (4.2).[3]

At the linearized level the gravitational potential in 4+1 dimensions with curved metric given by (2.3) obeys the following equation in four spatial dimensions

$$\nabla_3^2 V + \partial_z^2 V + 3\frac{d\Omega(\tilde{z})}{dz}\partial_z V = 4\pi^2 M G_5 \frac{\delta^{(3)}(\mathbf{x})\delta(z-z_1)}{e^{3\Omega}}\,. \tag{4.3}$$

This is nothing but (3.2), after we include a source term, due to a point particle of mass M located at the brane and at the origin of our three-dimensional spatial world. The normalization in the right-hand side of (4.3) is chosen such that for small distances the potential is $V = -\frac{G_5 M}{r^2+(z-z_1)^2}$, in accordance with (4.1). We have also dropped the time dependence since we are seeking static solutions. We may expand V in terms of the complete basis of eigenfunctions $\{h_Q\}$ of the operator $\partial_z^2 + 3d\Omega(\tilde{z})/dz\ \partial_z$ as

$$V = \sum_Q V_Q(r) h_Q(z)\,, \tag{4.4}$$

where the sum over Q comprises the massless as well as the massive modes. The general solution can be written formally as

$$V(r) = -\frac{\pi G_5 M}{r}\sum_{Q,P} e^{-M_Q r} K_{QP}^{-1} h_Q(z_1) h_P^*(z_1)\,, \tag{4.5}$$

[3]The model in [3], based on the AdS_5, has a continuous spectrum with no mass gap, and the corrections to Newton's law are power-like. They can be made extremely small when the curvature of the five-dimensional space-time is of Planck size [3].

with

$$K_{QP} = \int_{z_1}^{z_{\max}} dz e^{3\Omega(\tilde{z})} h_Q^*(z) h_P(z)\,, \tag{4.6}$$

with $z_{\max} = \infty$ and $\pi R/2$ for the case of our models (A) and (B). We have omitted the internal space dependence since all point particles are at z_1 with respect to our four-dimensional space-time. In general, we may assume that we have an orthonormal system of eigenfunctions $\{h_Q\}$. Then (4.5) becomes

$$V(r) = -\frac{G_4 M}{r}\left(1 + \sum_q d_q(z_1) e^{-M_q r}\right), \qquad d_q(z_1) \equiv |h_q(z_1)|^2 \int_{z_1}^{z_{\max}} dz e^{3\Omega}\,, \tag{4.7}$$

where the sum is now over the massive modes only and the four- and five-dimensional Newton constants are related as

$$G_4 = \frac{\pi G_5}{\int_{z_1}^{z_{\max}} dz e^{3\Omega}}\,. \tag{4.8}$$

The factor $d_q(z_1)$ in (4.7) weights the contributions of the various massive Kaluza–Klein states.[4] In practice it is difficult to construct an orthonormal basis, as can be seen from our examples (3.7) and (3.17). However, we may easily deduce that the general form of the Newton potential, with the leading correction included, will be

$$V(r) \simeq -\frac{G_4 M}{r}\left(1 + \alpha(r) e^{-r/\lambda}\right), \tag{4.9}$$

where the range λ of the Yukawa correction is related to the mass gap. The strength of the force $\alpha(r)$ could itself be a function of the distance r, as indicated, provided that the spectrum above the mass gap is continuous. However, if the spectrum is discrete the strength is just a constant. For the case of our examples we may show that

$$\text{model (A)}: \qquad \lambda = R\,, \qquad \alpha(r) = \text{const.}\left(\frac{R}{r}\right)^2, \tag{4.10}$$

and

$$\text{model (B)}: \qquad \lambda \sim R\,, \qquad \alpha(r) = \text{const.}\,, \tag{4.11}$$

where both constants appearing in the previous expressions depend on the position of the brane. For the choice of parameters in case I in section 2.1 both of these constants are of order 1, resulting in a measurable contribution to the strength of the Yukawa correction. Unlike this case, for the choice of parameters in case II in section 2.1 both of these constants are extremely small due to the wave function suppression.

[4] In fact (4.7) is very similar and extends the corresponding formulae proved in [18] for compactification on general compact manifolds (for the case of a torus compactification, see also [20]). One apparent difference is that, in the corresponding formulae in [18], the degeneracy of the irreducible representations of the symmetry group of the compact space appears instead of $d_q(z_1)$. However, as shown in [18], this degeneracy can be also written in terms of eigenfunctions of the scalar laplacean on the compact internal space.

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Hence, the corresponding Yukawa correction is negligible. We may demonstrate these different behaviours using the two orthonormal sets of wave functions (3.23) and (3.21) as they correspond to two extreme choices of the brane location.

For the special case of the orthonormal wave functions (3.23) with masses given by (3.24), the explicit computation can be carried out without much effort using properties of the Jacobi polynomials. We find that

$$
\begin{aligned}
V(r) &= -\frac{G_4 M}{r}\left(1+\sum_{n=1}^{\infty} A_n e^{-\frac{4r}{R}\sqrt{n(n+1/2)}}\right),\\
A_n &= \frac{4n+1}{2^{2n+1}n^2}\left(\frac{(2n-1)!!}{(n-1)!}\right)^2 .
\end{aligned}
\tag{4.12}
$$

In the limit of large r we may keep only the first term in the infinite sum above to a very good approximation

$$
V(r) \simeq -\frac{G_4 M}{r}\left(1+\frac{5}{8}e^{-\frac{2\sqrt{3}}{R}r}\right), \qquad \text{for} \quad r \gg R. \tag{4.13}
$$

In the opposite limit of small r we may approximate A_n in (4.12) by its value at infinity, i.e. $A_n \simeq A_\infty = 2/\pi$. Using also the relation $2\pi G_5 = RG_4$ (following from (4.8)) we indeed obtain the potential corresponding to Newton's law in four spatial dimensions:

$$
V(r) \simeq -\frac{G_5 M}{r^2}, \qquad \text{for} \quad r \ll R, \tag{4.14}
$$

as expected. The advantage of (4.12) is that the crossover behaviour from (4.14) to (4.13) is expressed in a simple and precise way.

For the special case of the orthonormal wave functions (3.21) with masses given by (3.22) we find that

$$
\begin{aligned}
V(r) &= -\frac{G_4 M}{r}\left(1+\sum_{n=1}^{\infty} B_n e^{-\frac{2r}{R}\sqrt{n(n+1)}}\right),\\
B_n &= \frac{(1-x_1)^3}{4}(2n+1)n^2(n+1)^2 + \mathcal{O}(1-x_1)^4 .
\end{aligned}
\tag{4.15}
$$

Hence, we see that using (2.22) and (3.16) all corrections to Newton's $1/r$ potential are $\mathcal{O}(10^{-66})$ and therefore negligible. As we have mentioned, this is due to the wave-function suppression near $x = 1$.

5. Discussion

In this paper we have considered models of our four-dimensional world in which a three-brane is embedded in five-dimensional backgrounds, which are solutions of gauged supergravity with a boundary term. These backgrounds arise as consistent truncations of solutions of type-IIB string theory, which correspond to continuous distributions of D3-branes on a disc or on a three-sphere. From the gauged supergravity point of view, the five-dimensional metrics are warped products of Minkowski space in four dimensions with an extra dimension. The interesting feature of our backgrounds is that the spectrum of linearized graviton fluctuations either has a genuine mass gap with a continuum above it or it is discrete. Furthermore, if the range of the fifth coordinate is reduced by placing a three-brane, a normalizable zero-mode appears, corresponding to the graviton in our world while the mass gap is preserved. Therefore, these models naturally lead to physics that appears four-dimensional, as long as energies are smaller than the mass gap, and an effective four-dimensional theory can be defined.

Furthermore, we studied in detail the corrections of Newton's law induced by the massive excitations that are Yukawa-like owing to the mass gap. We examined the space of parameters, under the condition that supergravity is a good approximation, for physically interesting scenarios. In one of them (case I in section 2.1) the fundamental scale is between the Planck and the electroweak energy scales and the radius of curvature of the space is of millimetre size. In this case the corrections are close to the experimental bounds and might be observed in future experiments; but still there exists a hierarchy, which, however, is smaller than the usual 10^{16}. In this range of parameters our models resemble those in [4] with a single compactified large extra dimension. The other scenario (case II in section 2.1) has the attractive feature that the fundamental scale is actually of the same order as the electroweak scale (unification) and it shares several features of compactification with three large extra dimensions, although our models have only one. The corrections to Newton's law, however, turn out to be negligible, because of a wave function suppression at the location of the three-brane. It would be very interesting to find models similar to the ones studied in this paper where the parameters can be chosen such that the corrections are closer to the experimental bounds whereas the fundamental scale can be of the order of TeV, i.e. we have unification.

Note added While we were finishing up our paper, [21] appeared, where the idea that the model of [2, 3] may correspond, with appropriate choice of parameters, to a direct product compactification with one large extra dimension, was also pointed out.

A. D3-branes distributions and gauged supergravity

The general ten-dimensional metric of an arbitrary distribution of parallel three-branes in type IIB string theory takes the following form

$$ds^2 = H^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(dr^2 + r^2 d\Omega_5^2)\,, \qquad \mu,\nu = 0,1,2,3\,, \tag{A.1}$$

where $d\Omega_5^2$ denotes the line element for the five-dimensional sphere and the function H is a harmonic function on the six-dimensional space transverse to the brane. One of the simplest examples is a stack of N coinciding branes in the near horizon limit: $H = R^4/r^4$ with $R^4 = 4\pi g_s N l_s^4$. In this case the background is $AdS_5 \times S^5$, which plays a central role in the AdS/CFT correspondence [10]. Upon reduction on the five-sphere, this yields $\mathcal{N} = 8$ gauged supergravity in five dimensions on AdS_5. The authors of [2, 3] utilized variants of this background by adding one or two three-branes, which cut off the fifth coordinate. In the full AdS_5 background, the gravity decouples from the four-dimensional world but, once the fifth coordinate does not run over the full range, gravity becomes dynamical. The AdS_5 solution in the bulk preserves thirty-two supersymmetries and is a stable background.

The metrics (A.1) with harmonic H corresponding to more general not-coinciding three-branes provide a large class of interesting backgrounds for compactification, which preserve sixteen supercharges. In the following we will concentrate on two backgrounds: (A) D3-branes distributed uniformly over a disc and (B) D3-branes distributed uniformly over a three-sphere. For (A) the metric can be written as [16]

$$ds^2 = \frac{r(r^2 + r_0^2 \cos^2\theta)^{1/2}}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu \; + \; \frac{R^2}{r(r^2 + r_0^2\cos^2\theta)^{1/2}} \times \tag{A.2}$$
$$\times \left((r^2 + r_0^2 \cos^2\theta)\left(\frac{dr^2}{r^2 + r_0^2} + d\theta^2\right) + (r^2 + r_0^2)\sin^2\theta d\phi^2 + r^2\cos^2\theta d\Omega_3^2 \right),$$

where $d\Omega_3^2$ is the line element of the three-sphere, r_0 is the radius of the disc and $r \geq 0$. The metric for the case (B) is obtained by taking $r_0 \to -ir_0$ and restricting the range of the radial coordinate $r \geq r_0$.

These two ten-dimensional backgrounds can be consistently truncated to five-dimensional gauged supergravity. More technical details can be found in [15]. The situation can be summarized as follows. The $\mathcal{N} = 8$ gauged supergravity in five dimensions contains forty-two scalars, which have a non-trivial potential. There is a stationary point where all scalars are zero (except for the complex coupling, which is a flat direction of the potential), which corresponds to the $AdS_5 \times S^5$ background in ten dimensions. For general D3-brane configurations the metric on S^5 is deformed, the ten-dimensional space becomes a warped product space and in the five-dimensional perspective (some of) the forty-two scalars develop non-trivial profiles. It is widely believed that every five-dimensional solution can be lifted unambiguously to ten dimensions, but a complete proof is still missing. There exist privileged flows such that the scalars lie in a one-dimensional submanifold of the forty-two scalars, which we will denote by a scalar ϕ. It was shown in [15] that these solutions correspond to certain D3-brane distributions in ten dimensions – including our two examples.

The relevant part of the bosonic action of the five-dimensional gauged supergravity, in $-++++$ signature, is

$$\mathcal{L} = \sqrt{-G}\left(\frac{1}{4}\mathcal{R} - \frac{1}{2}G^{MN}\partial_M\phi\partial_N\phi - P(\phi)\right), \tag{A.3}$$

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where ϕ is the scalar field with potential P, which can be expressed in terms of an auxiliary function W

$$P = \frac{1}{2R^2}\left(\frac{\partial W}{\partial \phi}\right)^2 - \frac{4}{3R^2}W^2\,. \tag{A.4}$$

The five-dimensional metric has the form

$$ds^2 = e^{2\Omega(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\,. \tag{A.5}$$

If the following two conditions:

$$\frac{d\phi}{dz} = \frac{1}{R}e^{\Omega}\frac{\partial W}{\partial \phi}\,, \qquad \frac{d\Omega}{dz} = -\frac{2}{3R}e^{\Omega}W\,, \tag{A.6}$$

are obeyed, then the solution preserves sixteen supercharges. Solutions of (A.6) automatically fulfil the field equations of (A.3).

We summarize here the results for our two examples [16, 15]. For D3-branes smeared over a disc (case (A)) we have

$$\begin{aligned} W &= e^{2\phi/\sqrt{6}} + \frac{1}{2}e^{-4\phi/\sqrt{6}}\,, \\ \Omega &= -\frac{1}{2}\ln\left|e^{-2\phi/\sqrt{6}} - e^{4\phi/\sqrt{6}}\right| + \ln\left(\frac{r_0}{R}\right), \\ P &= -\frac{1}{R^2}\left(e^{4\phi/\sqrt{6}} + 2e^{-2\phi/\sqrt{6}}\right), \end{aligned} \tag{A.7}$$

where ϕ is a scalar field depending on z. The functions W, Ω and the potential P for the case (B) can be obtained by replacing $\phi \to -\phi$. Equation (A.6) can be solved for ϕ. In the case (A) the solution is

$$\phi = \sqrt{\frac{2}{3}}\ln\cosh\left(\frac{zr_0}{R^2}\right), \tag{A.8}$$

while in case (B) it is

$$\phi = \sqrt{\frac{2}{3}}\ln\cos\left(\frac{zr_0}{R^2}\right). \tag{A.9}$$

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